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ON ESTIMATES OF THE ATMOSPHERIC ENERGY CYCLE

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ABSTRACT

Several estimates of the rate at which the mean and eddy forms of both kinetic energy and available potential energy are generated, converted, and dissipated in the atmosphere are compared in tabular form. From these tables a selection is made of those values which are, in the author's opinion, representative for the yearly energy cycle in the Northern Hemisphere.

1. INTRODUCTION

In this article a critical survey will be given of statistics obtained by several investigators for the large-scale generation, dissipation, and conversion of energy in the atmosphere. Kinetic and available potential energy, both subdivided into their mean and eddy (deviations from the mean) contributions, will be considered as four separate forms of energy. This formulation was introduced by Lorenz [8].

The discussion will be restricted to the Northern Hemisphere. Only long time averages (i.e., averages over a period of a year, a winter or a summer half-year) will be considered; this is often a necessary condition for obtaining statistically significant values.

Only rough agreement is found among the estimates of different investigators regarding the fundamental characteristics of the general circulation. Therefore, it is of interest to examine the available material for the energy budget, and to attempt to construct from this material a quantitative picture of the energy cycle.

2. POSSIBLE DEFINITIONS OF MEAN AND EDDY FIELDS

If one considers the atmosphere, one can distinguish among three classes of eddies, namely eddies in the zonal, the meridional, and the vertical direction. We shall consider only the eddies in the zonal direction and also the eddies in time.¹ Thus, the eastward and northward components of the wind u and v, and the temperature T may be written as the sum of four components

$$u = [\overline{u}] + \overline{u}^* + [u]' + u'^*$$

$$v = [\overline{v}] + \overline{v}^* + [v]' + v'^*$$

$$T = [\overline{T}] + \overline{T}^* + [T]' + T'^*$$

where the brackets represent a zonal average, the star a deviation from the zonal average, the bar a time average, and the prime a deviation from the time average. The kinetic and potential energy connected with the four components will be denoted by, respectively, the subscripts 0, 1, 2, and 3. We shall be interested mainly in three methods of separating the total kinetic and available potential energy into their mean and eddy parts. Let us define these methods as follows:

(1) Space domain:

$$K_{M} = K_{0} + K_{2} = \frac{1}{2} \int \overline{([u]^{2} + [v]^{2})} dm$$

="zonal kinetic energy" in Lorenz [8] paper.

$$K_E = K_1 + K_3 = \frac{1}{2} \int \overline{([u^{*2}] + [v^{*2}])} dm$$

="eddy kinetic energy" in Lorenz [8] paper.

¹ For a discussion of the exchanges between the vertical mean field and the vertical eddy field see Wiin-Nielsen [29] and Smagorinsky [20].

$$P_{M} = P_{0} + P_{2} = \frac{1}{2} c_{p} \int \sqrt{|T|''^{2}} dm$$

= "zonal available potential energy" in Lorenz [8] paper. (A double prime represents a deviation from the area average over a closed pressure surface)

$$P_{E} = P_{1} + P_{3} = \frac{1}{2} \int \sqrt{\overline{\gamma}[T^{*2}]} dm$$

="eddy available potential energy" in Lorenz [8] paper.

(2) Time domain:

$$\mathbf{K}_{M} = K_{0} + K_{1} = \frac{1}{2} \int [\overline{u}^{2} + \overline{v}^{2}] dm$$

$$\mathbf{K}_{E} = K_{2} + K_{3} = \frac{1}{2} \int [\overline{u'^{2}} + \overline{v'^{2}}] dm$$

$$\mathbf{P}_{M} = P_{0} + P_{1} = \frac{1}{2} c_{p} \int \gamma [\overline{T''^{2}}] dm$$

$$\mathbf{P}_{E} = P_{2} + P_{3} = \frac{1}{2} c_{p} \int \gamma [\overline{T'^{2}}] dm$$

(3) Mixed space-time domain:

$$\begin{split} \mathcal{K}_{M} &= K_{0} = \frac{1}{2} \int ([\overline{u}]^{2} + [\overline{v}]^{2}) \, dm \\ \mathcal{K}_{E} &= K_{1} + K_{2} + K_{3} = \frac{1}{2} \int [\overline{u'^{2}} + \overline{v'^{2}} + \overline{u}^{*2} + \overline{v}^{*2}] \, dm \\ \mathcal{P}_{M} &= P_{0} = \frac{1}{2} \, c_{p} \int \gamma [\overline{T}]^{\prime\prime \, 2} \, dm \\ \mathcal{P}_{E} &= P_{1} + P_{2} + P_{3} = \frac{1}{2} \, c_{p} \int \gamma [\overline{T^{\prime\prime \, 2}} + \overline{T}^{*2}] \, dm \end{split}$$

All integrals are taken over the mass of the entire atmosphere. In these expressions

(")=area average over a closed pressure surface,
(")"=deviation from this area average,

 c_p =specific heat at constant pressure, and

$$\gamma = -\left(\frac{\Theta}{T}\right)^2 \frac{R}{c_p p_0} \int_0^{p_0} \left(\frac{T}{\Theta}\right) \frac{1}{p} \left(\frac{\partial \widetilde{\Theta}}{\partial p}\right)^{-1} dp,$$

 $\Theta =$ potential temperature, R = gas constant, and $p_0 =$ 1000 mb. (Lorenz used in this case $\gamma = -\frac{\Theta}{T} \frac{R}{c_r p} \left(\frac{\partial \widetilde{\Theta}}{\partial p}\right)^{-1}$. However, if one uses this last expression for γ , extra terms will enter the equations for the balance of available potential energy aside from the terms which represent the expected energy transformations. These extra terms depend mainly upon the variation of the mean (i.e., averaged over a closed pressure level) static stability with pressure. In order to avoid these extra (and probably meaningless) terms, it appeared necessary to use from the beginning $\gamma = -k \left(\frac{\Theta}{T}\right)^2$ in the definition of available po-

tential energy, where k is independent of pressure and time; see e.g., the expression given above. The numerical differences with Lorenz's formulae are quite small.)

In the space domain mean kinetic energy is defined as the kinetic energy of the zonally averaged motion, in the time domain it is defined as the kinetic energy of the time-mean motion, and in the mixed space-time domain as the kinetic energy of the time-mean and zonal-mean motion. What is left of the total kinetic energy is called in each case "eddy" kinetic energy. Analogous definitions are used in the case of mean and eddy available potential energy. One should clearly distinguish between these three methods, since it is obvious that the values of the mean kinetic and mean available potential energy computed in the space and time domain are both larger than those computed in the mixed space-time domain. On the other hand, eddy kinetic and eddy potential energy computed in the space and time domain are smaller than if they were calculated in the mixed space-time domain. Also, the numerical values of the rate at which mean or eddy energy is generated, dissipated, and transformed, are quite different in the three systems. Knowledge of the energy cycle in the space domain answers questions concerning the maintenance of the zonal-mean state and the zonal eddies. On the other hand, a similar knowledge in the time domain enables us to consider the maintenance of the time-mean state and the transient (in time) eddies. Finally, the energy cycle in the mixed domain gives information on how the time- and zonal-mean state and the sum of transient and standing eddies are maintained. The energy integrals for the three systems will be compared in more detail in the appendix of this paper. In general, it is difficult to decide which one of the three approaches is preferable. The choice must depend more or less on the information which one desires to obtain.

3. DATA AND METHODS OF COMPUTATION

A separation into three categories will be made according to the kind of data and the methods of reduction used in calculating the energy integrals in tables 1, 2, 3, and 4.

- (A) Basic information consists of the observed values of the horizontal wind components and the temperature. Vertical motions are computed with the so-called adiabatic method. Especially over the ocean areas this method is hampered by inadequate data coverage. Principal investigators: V. P. Starr, H. S. Buch, E. Holopainen, C. E. Jensen, J. P. Peixoto, and R. M. White.
- (B) Basic information consists of height data of the isobaric levels, which are obtained from the objective, daily analyses by the National Meteorological Center (NMC) of the U.S. Weather Bureau. Vertical motions are computed from a two- or three-level model using again only height data; in general, effects of heating and friction are neglected. Method B allows for daily estimates of winds over data-sparse regions; further, the objective analyses are aided by extrapolation from sea

level observations and also by reports from reconnaissance aircraft. A drawback of method B is the geostrophic assumption which plays an essential role in this approach. On the other hand, method B can supply valuable indirect (i.e., through a model) information on, for example, the heating rates in the atmosphere, and thereby also on the generation of available potential energy, where direct information is not yet available. Principal investigators: A. Wiin-Nielsen, J. E. Brown, B. Saltzman, A. Fleisher, A. F. Krueger, J. S. Winston, and D. Haines.

(C) Basic input consists of the hydrodynamical equations with some necessary simplifications and suitable boundary conditions. No actual data are used at all. In such numerical experiments one attempts among other things to approximate the essential features of the general circulation as they are determined under A and B. Principal investigators: N. A. Phillips, J. Smagorinsky, and Y. Mintz.

Estimates of the terms in the energy cycle are available only in the "space" and in the "mixed space-time" domain. Most computations have been carried out in what was called above the space domain. In this method all integrals are computed on a daily basis, while the averaging in time occurs only as a last step. As far as is known to the author, no extensive calculations have been carried out in the time domain. On the other hand, some work has been done in the mixed space-time domain. In this method the evaluation of the energy transformations is less time-consuming than in the space or time domain since it is not necessary to calculate these integrals for each day separately. In the next section we shall present some tables which give the estimates of several investigators of the energy integrals in the space and mixed space-time domain.

The following symbols will be used:

 K_M, K_E =mean, eddy kinetic energy in the space domain P_M, P_E =mean, eddy available potential energy in the space domain

 $\mathbf{K}_{M}, \mathbf{K}_{E}$ =mean, eddy kinetic energy in the time domain $\mathbf{P}_{M}, \mathbf{P}_{E}$ =mean, eddy available potential energy in the time domain

 $\mathcal{K}_{\scriptscriptstyle{M}}, \mathcal{K}_{\scriptscriptstyle{E}} = \text{mean, eddy kinetic energy in the mixed space-time domain}$

 $\mathcal{P}_{\scriptscriptstyle{M}}, \mathcal{P}_{\scriptscriptstyle{E}} = \text{mean}$, eddy available potential energy in the mixed space-time domain

G(P)=rate of generation of available potential energy by diabatic heating

D(K)=rate of frictional dissipation of kinetic energy $C(P_M,K_M)$ =rate of conversion from mean available potential energy into mean kinetic energy by mean meridional circulations

 $C(P_E, K_E)$ =rate of conversion from eddy available potential energy into eddy kinetic energy by large-scale eddy convection

 $C(K_E,K_M)$ =rate of conversion from eddy into mean kinetic energy by eddy momentum transport

 $C(P_M, P_E)$ = rate of conversion from mean into eddy available potential energy by eddy heat transport.

4. DISCUSSION OF ENERGY INTEGRALS

In tables 1 and 2 some estimates are given of the amount of mean and eddy kinetic and available potential energy present in the atmosphere of the Northern Hemisphere. The estimates in the space domain are presented in table 1 and those in the mixed space-time domain in table 2. The tables contain, besides the estimates, some relevant information concerning the representativeness of the data.

Estimates of the energy generation, dissipation, and conversion in the space and mixed space-time domain may be found in tables 3 and 4. Some comments will be made concerning the energy integrals in these tables.

(1) Generation of eddy available potential energy takes place if at a certain pressure level relatively warm. air masses are heated and cold air masses are cooled as a result of the combined effects of radiation, condensation or evaporation, and turbulent exchanges with the earth's surface. This eddy generation was computed in the space domain by Wiin-Nielsen and Brown [30] and Brown [1] with the aid of daily hemispheric heating fields which were calculated from the thermodynamic equation. According to their calculations diabatic heating destroys eddy available potential energy. This could be expected if the exchange of heat between the atmosphere and the earth's surface were the determining factor; in general this exchange will have the tendency to cool warmer air masses and to warm colder air masses. The 2-parameter baroclinic model used by Wiin-Nielsen and Brown for evaluating vertical motions may underestimate the creation of eddy available potential energy due to condensation heating. This would make their eddy generation too negative. Suomi and Shen [26] found a buildup of eddy potential energy which was due only to the influence of infrared cooling. This was derived from radiation measurements made by Explorer VII. The sample was limited to a period of 13 days and to a small horizontal area. Therefore, it is probably not representative and further measurements are needed.

From balance considerations it follows that in the mixed space-time domain not a destruction but a creation of eddy available potential energy may occur (see next section).

- (2) The dissipation of kinetic energy by friction cannot be measured directly. The best method is to compute the dissipation as the residual term in the balance of kinetic energy. This was done for the sum of mean and eddy kinetic energy by Holopainen [5] for each day of a period of three winter months over England. Although the daily values cannot be trusted, the order of magnitude might be correct.
- (3) Overturnings of the atmosphere on the largest scale (i.e., rings of air moving across latitude circles) give a con-

Table 1.—Estimates of the average amount of energy (105 joule m.-2)* per unit area in the Northern Hemisphere.**
in the "space domain" from height data only Computed

Investigators	Saltzman [18, 19], Fleisher [18, 19]	Teweles [27]
Representative for area covering	20-80° N. for P _M , P _E 15-80° N. for K _M , K _E	17.5°-77.5° N.
P_M	53.0 6 winter mo. 1959; 850– 500-mb. thickness [18]	
P_B	13.9 6 winter mo. 1959; 850– 500-mb thickness [18]	
K_{M}	6.3 year 1951; 500-mb. height data [19]	14.4 3 winter mo. 1957–1958; 500-, 100-, 50-mb. height data [27]
	8.9 winter 1951; 500-mb. height data [19] 3.7 summer 1951; 500-mb. height data [19]	aada [21]
<i>K_B</i>	7.8 year 1951; 500-mb. height data [19] 9.8 winter 1951; 500-mb. height data [19] 5.8 summer 1951; 500-mb. height data [19]	8.1 3 winter mo. 1957-1958; 500-, 100-, 50-mb. height data; wave numbers 1-8 [27]

*105 joule m.-2=105 erg cm.-2 is equivalent with 2.56×1026 erg for the atmosphere of the entire Northern Hemisphere.

**Values are integrated in vertical direction throughout the depth of the atmosphere. October, November, December, January, February, and March are considered as "winter" months. April, May, June, July, August, and September are considered as "summer" months.

version between mean potential and mean kinetic energy. This term is not important in the hemispheric energy balance; see in this connection Starr [22, 23], Wiin-Nielsen [28], Saltzman and Fleisher [17, 18]. Starr used the expression $f(\bar{u})[\bar{v}] dm$, where u, v = west-east, south-north component of the actual wind and f=Coriolis parameter. to compute the conversion from mean potential into mean kinetic energy in the mixed domain. This conversion is usually given by the expression $-\int [\overline{\omega}] [\overline{\alpha}] dm$, where $\omega =$ dp/dt="vertical velocity" and α =specific volume. The two expressions are identical if u is replaced by u_s , the geostrophic component of u. Krueger, Winston, and Haines [7] using the new NMC 3-parameter model for computing vertical velocities, obtained large negative values for this conversion in the space domain, the reason being that they did not include the area south of 20° N. in their integration. If one integrates over the entire hemisphere, the influence of the direct Hadley circulation at low latitudes should bring these estimates closer to zero. The values obtained by Wiin-Nielsen [28] and Saltzman and Fleisher [17, 18] also for the area north of 20° N. are not so large negatively probably because of the use of the NMC 2-parameter vertical velocities which appear to give smaller conversions between potential and kinetic energy. Summarizing, our best estimate is that the conversion between mean potential and mean kinetic energy is small compared to, for example, the conversion from eddy potential into eddy kinetic energy.

(4) The eddy conversion from potential into kinetic energy in the space domain is given essentially by the covariance of ω and T within latitude circles. In all estimates of ω which were used for the calculation of $C(P_E, K_E)$ in table 3, the effects of diabatic heating have

Table 2.—Estimates of the average amount of energy (105 joule m.-2)* per unit area in the Northern Hemisphere.**
"mixed space-time domain"

	A. From actual wind and temperature data	B. From height data only		
Investigators	Buch [3], Crutcher [4], Mura- kami [9], Peixoto [12]	Saltzman [19], Fleisher [19]		
Representative for area covering	Northern Hemisphere (10-70° N. for [9])	15–80° N.		
$\mathscr{P}_{\scriptscriptstyle M}$	26.4 year 1950: temperature data at 7 levels (computed from [12])			
$\mathscr{P}_{\scriptscriptstyle E}$	14.7 year 1950; temperature data at 7 levels (computed from [12])			
$\mathcal{K}_{\scriptscriptstyle M}$	3.5 5 years; winds at 6 levels (computed from [4]) 3.2 year 1950; winds at 6 levels (computed from [3] by [9])	5.2 year 1951; 500-mb. height data (computed from [19]) 8.3 winter 1951; 500-mb. height data (computed from [19]) 2.9 summer 1951; 500-mb. height data (computed from [19])		
$\mathcal{K}_{\scriptscriptstyle E}$	9.5 year 1950; winds at 6 levels (computed from [3] by [9])	8.9 year 1951; 500-mb. height data (computed from [19]) 10.4 winter 1951; 500-mb. height data (computed from [19]) 6.6 summer 1951; 500-mb. height data (computed from [19])		

 ^{**} See footnotes table 1.

been neglected. Unfortunately, it is not known how reliable this approximation is. The same difficulty holds in the computations of the conversion in the mixed space-time domain.

- (5) The horizontal area of integration does not always cover the entire hemisphere. For example, the objective NMC analyses which were used by the investigators of group B do not extend to latitudes lower than 17.5° N. For this reason the conversion and generation terms are integrated only over an area from the pole to about 20° N. It would, indeed, be incorrect to extrapolate these conversion rates to the equator. Recently indications have been found of a negative value of the rates of conversion from mean potential into eddy potential and from eddy potential into eddy kinetic energy at low latitudes (in middle latitudes these integrals are large and positive). In other words, there is evidence of an eddy heat transport against the mean meridional temperature gradient and also of a conversion from kinetic into potential energy by the large-scale eddy processes in tropical latitudes (see Peixoto [11], Starr and Wallace [24]). This indirect action of the eddies in the Tropics may be compared with the quite similar operation of the disturbances in the lower stratosphere (see Oort [10]).
- (6) The rate of transformation between mean and eddy kinetic energy is given essentially by the product of the eddy transport of momentum and the gradient of mean angular rotation both taken in the north-south direction. Similarly, the rate of transformation between mean and

Table 3.—Estimates of the energy integrals (watt m.-2)* in the "space domain" for the Northern Hemisphere**

	A. From actual wind and temperature data	B. From height data on puted from a fricti	ly (geostrophic approach); onless, adiabatic model usi	vertical motions are com- ng only height data		lution hydrodynamical ations
Investigators	Starr [21, 23, 25], Brunt [2], Holopainen [5,] Suomi [26], Shen [26], White [25]	Saltzman [15, 16, 18,], Fleisher [16, 18]	Wiin-Nielsen [28, 31], Brown [1, 31], Drake [31]	Krueger [7], Winston [7], Haines [7], Teweles [27]	Phillips [14]; 2-level quasi- geostrophic model	Smagorinsky [20]; 2-level model using primitive equations
Representative for area cover- ing	Northern Hemisphere	Northern Hemisphere	20-90° N.	20-90° N.	Rectangular region: -5000 km.≤y≤5000 km. 0≤x≤6000 km.	0-64.4° N.
$G(P_M)$			1.94 year (9 mo. 1959-63); 850, 500-mb. height data and model for heating [1] 2.81 7 winter mo. 1959-63; 850, 500-mb. height data and model for heating [1] 1.07 6 summer mo. 1961-62; 850, 500-mb. height data and model for heating [1]	2.32 year; computed as residual term [7] 3.30 winter; computed as residual term [7] 1.34 summer; computed as residual term [7]	2.13 heating minus lateral heat diffusion	2.21 heating minus lateral heat diffusion
$G(P_R)$	0.58 13 days 1959-1960; infrared cooling measured from Explorer VII; 30-50° N.; sample is probably not representative for atmosphere [26]		-0.94 year (9 mo. 1959-63); 850, 500-mb. height data and model for heat- ing [1] -1.57 winter mo. 1959-63; 850, 500- mb. height data and model for heating [1] -0.32 6 summer mo. 1961-62; 850, 500- mb. height data and model for heating [1]	-0.77 year; computed as residual term [7] -1.10 winter; computed as residual term [7] -0.44 summer; computed as residual term [7]	-0.10 lateral heat diffusion	-0.28 heating minus lateral heat dif- fusion.
$D(K_M)$	-5.0 mean + eddy dissipation based on mean wind profile; extreme-	-0.23 winter; computed as residual term [15]	::		-0, 95 skin friction plus effects of lateral eddy viscosity taken into ac- count	-1.25 skin and internal friction plus ef- fects of lateral eddy viscosity taken into ac- count
$D(K_R)$	fy uncertain [2] mean + eddy dissipation; Sept., Oct., Nov. 1954; area England; com- puted as resid- ual term on a daily basis [5]	-2.37 winter; computed as residual term [15]			-0.89 skin friction plus effects of lateral eddy viscosity taken into ac- count	-1.50 skin and internal friction plus ef- fects of lateral eddy viscosity taken into ac- count
$C(P_M, K_M)$	0.25 year 1950; winds at 7 levels; analy- sis by strings of stations (comput- ed from [25])	0.35 6 winter mo. 1959; 850-500-mb, thick- ness and \(\omega \) at 600 mb.; includes by extrapolation con- tribution of equa- torial Hadley cell [18]	0.10 Jan. 1959; 850-500-mb, thickness and ω at 600 mb, [28] -0.11 Apr. 1959; 850-500-mb, thickness and ω at 600 mb, [28]	-0.66 year 1962-63; 3-parameter NMC model for ω [7] -0.79 6 winter mo 1962-63; 3-parameter NMC model for ω [7] -0.53 6 summer mo. 1962-63; 3-parameter NMC model for ω [7]	-0.39	-0.10
$C(P_B, K_E)$		3.02 6 winter mo. 1959; area 20-80° N; 850- 500-mb. thickness and \(\omega \) at 600 mb. [18]	1.46 Jan. 1959; 850-500-mb. thickness and	2.21 year 1962-63; 3- parameter NMC model for ω [7] 2.98 6 winter mo. 1962- 63; 3-parameter NMC model for ω [7] 1.44 6 summer mo. 1962-63; 3-pa- rameter NMC model for ω [7]	3.47	2, 44
$C(K_B, K_M)$	0.38 first 6 mo. 1950; winds at 5 levels; analysis by strings of stations in a latitude belt [21] 0.23 year 1951; winds at 7 levels; analysis by strings of stations in a latitude belt [23] 0.25 year 1951; only 500-mb. winds; analysis by strings of stations in a latitude belt [23]	0.15 year 1951; only 500-mb. height data [16] 0.23 6 winter mo. 1951; only 500-mb. height data [16] 0.07 6 summer mo. 1951; only 500-mb. height data [16]	0.16 year (5 mo. 1962-63); height data at 5 levels [31] -0.08 4 winter mo. 1962-63; height data at 5 levels [31] 0.40 3 summer mo. 1962; height data at 5 levels [31]	0.14 IGY (5 mo.); area 17.5-77.5° N; 500, 100, 50-mb, height data [27]	1.48	1.20
$C(P_M, P_B)$			3.25 year (5 mo. 1962– 63); height data at 5 levels [31] 4.54 4 winter mo. 1962– 63; height data at 5 levels [31] 1.96 3summer mo. 1962; height data at 5 levels [31]	2.98 year (60 mo. 1958– 63); 850, 500-mb. height data [7] 4.08 30 winter mo. 1958– 63; 850, 500-mb. height data [7] 1.88 30 summer mo. 1958–63; 850, 500- mb. height data [7]	3.47	2.87

^{*1} watt m.-2=103 erg cm.-2 sec.-1 is equivalent with 2.56×10^{21} erg sec.-1 for the atmosphere of the entire Northern Hemisphere.
**Values are integrated in vertical direction throughout the depth of the atmosphere.

October, November, December, January, February and March are considered as "winter" months. April, May, June, July, August and September are considered as "summer" months.

Table 4.—Estimates of the energy integrals (watt m.-2)* in the "mixed space-time domain" for the Northern Hemisphere**

FROM ACTUAL WIND AND TEMPERATURE DATA					
Investigators	Starr [21, 23, 25], White [25]	Brunt [2], Buch [3], Holo- painen [5], Jensen [6], Peixoto [12, 13]			
Representative for area covering	Northern Hemisphere	Northern Hemisphere			
$G(\mathscr{P}_{\scriptscriptstyle{M}})$	1.12 year 1950 (computed as a residual term from [23, 25])				
$G(\mathscr{P}_{\scriptscriptstyle{B}})$					
$D(\mathcal{K}_{\scriptscriptstyle{M}})$	-0.10 year 1950 (computed as a residual term from [21, 23])	-5.0 mean + eddy dissipa- tion based on mean wind profile; extreme- ly uncertain [2]			
$D(\mathcal{K}_{\scriptscriptstyle{B}})$		-1.9 mean + eddy dissipation; Sept., Oct., Nov. 1954; area England; computed as residual term on a daily basis [5]			
$C(\mathcal{P}_{\scriptscriptstyle{M}},\mathcal{K}_{\scriptscriptstyle{M}})$	-0.08 year 1950; winds at 7 levels; analysis by strings of stations [23] -0.12 year 1951; winds at 7 levels; analysis by strings of stations [23]				
$C(\mathcal{P}_{\scriptscriptstyle{\mathcal{B}}},\mathcal{K}_{\scriptscriptstyle{\mathcal{B}}})$		$ \begin{array}{llllllllllllllllllllllllllllllllllll$			
$C(\mathcal{K}_{\scriptscriptstyle B},\mathcal{K}_{\scriptscriptstyle M})$	0.15 first 6 mo. 1950; winds at 5 levels; analysis by strings of stations in a latitude belt [21] 0.21 second 6 mo. 1950; winds at 5 levels; analysis by strings of stations in a latitude belt [21]	0.22 year 1950; winds at 6 levels; analysis of maps; transient and standing eddies included (com- puted from [3])			
$C(\mathcal{P}_{\scriptscriptstyle{M}},\mathcal{P}_{\scriptscriptstyle{B}})$	1.20 year 1950; winds and temperatures at 7 levels; analysis by strings of stations in a latitude belt (computed from [25]).	0.96 year 1950; winds and temperatures at 7 levels; analysis of maps; only transient eddies included (computed from [12]) 2.02 6 winter mo. 1950; winds and temperatures at 7 levels; analysis of maps; transient and standing eddies included [13] 0.71 6 summer mo. 1950; winds and temperatures at 7 levels; analysis of maps; transient eddies only (computed from [12])			

^{* **}See footnotes table 3.

eddy potential energy is given by the product of the eddy transport of heat and the gradient of mean temperature, also both in the north-south direction. For both terms there is a considerable numerical difference between the conversion rates computed in the space and in the mixed space-time domain. As an illustration, we may quote Starr's [21] measurements of the conversion from eddy into mean kinetic energy for the first six months of 1950. Starr computes 0.38 watt m.⁻² in the space domain and with the same data 0.15 watt m.⁻² in the mixed space-time domain. There is evidence of a similar difference in the rate of conversion from mean into eddy potential energy. The investigators of group B, using Lorenz's approach,

- estimate a value of about 3.0 watt m.⁻² for the year, but the data of the investigators of group A, using the mixed space-time approach, give the much smaller value of about 1.2 watt m.⁻². The rate of conversion from mean into eddy potential energy is stronger in the space than in the mixed domain, since in the first case an extra term is included which depends on the covariance in time of the down-gradient eddy transport of heat and the meridional temperature gradient itself (see appendix). As will be seen in the next section, this extra term may significantly alter the value of the generation of eddy available potential energy as determined from balance conditions.
- (7) Starr [23] noticed that the integral $C(K_E, K_M)$ had a positive value for each month of the year 1951; i.e., during each month the eddies gave kinetic energy to the zonal current. The same was true for four of the five months considered by Wiin-Nielsen, Brown, and Drake [31]; only January 1963 formed an exception. During this month the large-scale disturbances drew kinetic energy from the zonal flow. In this respect January 1963 must have been an exceptional month.
- (8) A comparison of the energy integrals calculated in the numerical experiments (Phillips [14], Smagorinsky [20]) with those calculated from actual data shows that the dissipation of eddy kinetic energy in the experiments is too small, while the dissipation of mean kinetic energy is much too large. Presumably this is related to difficulties in formulating the dissipation mechanism. A further consequence of the incorrect dissipation rates is that too much eddy kinetic energy is converted into mean kinetic energy.

5. DIAGRAM FOR THE ATMOSPHERIC ENERGY CYCLE

From the generation, dissipation, and conversion rates in tables 3 and 4 a careful selection was made by considering the representativeness of the data. The author's best estimates of the order of magnitude of the hemispheric energy processes for the period of a year are presented in two diagrams, one for the space domain (fig. 1) and the other for the mixed space-time domain (fig. 2). No special averaging process was used in determining the numbers from the data given in tables 1 through 4. The necessary balance conditions for each energy component have been taken into account in the construction of these flow diagrams. In the diagrams the small circles indicate the four forms of energy. The large circle which encloses the four smaller ones represents the "environment". The numbers on the connecting lines indicate the rate of conversion, generation, or dissipation for the different forms of energy. The numbers within the small circles give the amount of energy present. The dissipation rates are derived indirectly from the other estimates using balance requirements; the same holds to a certain extent for the generation rates.

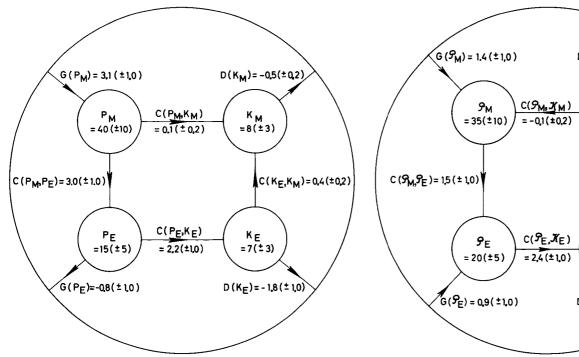


FIGURE 1.—Tentative flow diagram of the atmospheric energy in the space domain. Values are averages over a year for the Northern Hemisphere. Energy units are in 105 joule m.-2 $(=10^8 \text{ erg cm.}^{-2});$ energy transformation units are in watt m. $^{-2}$ $(=10^3 \text{ erg cm.}^{-2} \text{ sec.}^{-1}).$

 $D(\mathcal{K}_{M}) = -0.2(\pm 0.2)$ ХM C(KE,KM)=0.3(±0.2) χE =10(±3) $D(\mathcal{K}_{E}) = -2.1(\pm 1.0)$

FIGURE 2.—Tentative flow diagram of the atmospheric energy in the mixed space-time domain. Values are averages over a year for the Northern Hemisphere. Energy units are in 10⁵ joule m.⁻²; energy transformation units are in watt m -2

As is well known the energy cycle proceeds in the average from mean available potential energy via eddy available potential energy and eddy kinetic energy, to, finally, the mean kinetic energy, i.e., in a counterclockwise direction in figures 1 and 2. The important steps in this cycle will be discussed below in some detail. The numbers for the mixed space-time domain will be placed in parentheses after the numbers for the space domain.

At low latitudes the earth-atmosphere system gains more energy per unit area by the absorption of shortwave radiation from the sun than it loses to space by the emission of long-wave radiation; the reverse is true at high latitudes. Since in the troposphere, which contains the bulk of the atmosphere, the higher temperatures are found at low latitudes and the lower temperatures at high latitudes, mean available potential energy is created as a result of radiation. The creation due to all diabatic sources amounts to 3.1 (1.4) watt m.⁻². Without rotation the release of this energy would take place through an axially symmetric, meridional Hadley circulation. However, because of the actual conditions of rotation and heating of the earth and its atmosphere, asymmetric, eddy circulations take over the task of the mean meriddional circulation as convective units. The zonal currents connected with the meridional temperature gradient have a maximum instability at the wavelength of the

large-scale eddies. The conversion from mean available potential energy into its eddy counterpart occurs at the estimated rate of 3.0 (1.5) watt m.⁻². In the space domain heating processes appear to destroy (in the mixed domain they appear to create) eddy potential energy at the rate of -0.8 (+0.9) watt m.⁻². The remaining energy, i.e., 2.2 (2.4) watt m.⁻², is converted into eddy kinetic energy. The conversion from eddy kinetic into mean kinetic energy, which maintains the zonal currents against the frictional dissipation by small-scale eddies, is only about 0.4 (0.3) watt m.-2. To maintain the balance of the eddy kinetic energy it is necessary that about 80 percent of the eddy kinetic energy which is released by convection, i.e., -1.8 (-2.1) watt m.⁻², be destroyed by friction. The dissipation of mean kinetic energy is comparatively small, namely -0.5 (-0.2) watt m.⁻², since the exchange between mean kinetic and mean available potential energy is only 0.1 (-0.1) watt m. $^{-2}$.

APPENDIX

Expressions for the energy integrals in the space (a), time (b), and mixed space-time (c) domain will be presented. Further, in order to obtain a better understanding of the differences in the flow diagrams (figs. 1 and 2) for the energy cycle, the integrals for the space and mixed domains will be compared (d).

NOTATION

 λ , $\phi =$ longitude, latitude

p = pressure

u, v = eastward, northward components of the wind

 $\omega = dp/dt =$ "vertical velocity"

Φ=geopotential "height"

T=temperature

 Θ =potential temperature

dm =increment of mass

a = radius of the earth

f=Coriolis parameter

R = gas constant

 c_p = specific heat at constant pressure

Q=rate of heat addition per unit mass

F=frictional force per unit mass

$$\gamma = - \left(\frac{\Theta}{T}\right)^2 \frac{R}{c_p p_0} \int_0^{p_0} \left(\frac{T}{\Theta}\right) \frac{1}{p} \left(\frac{\partial \widetilde{\Theta}}{\partial p}\right)^{-1} dp$$

(see discussion section

 $p_0 = 1000 \text{ mb}.$

 \overline{b} = time average of b

b' = deviation from time average of b

[b] = zonal average of b

 b^* =deviation from zonal average of b

 \tilde{b} = hemispheric average of b over isobaric surface

 $b^{\prime\prime}$ = deviation from hemispheric average of b.

The basic equations used in deriving the formulae for the generation, destruction, and conversion rates of kinetic and available potential energy are the first law of thermodynamics

$$\frac{\partial\Theta}{\partial t} = -u \frac{\partial\Theta}{a\cos\phi\partial\lambda} - v \frac{\partial\Theta}{a\partial\phi} - \omega \frac{\partial\Theta}{\partial\rho} + \frac{1}{c_p} \left(\frac{\Theta}{T}\right)Q \qquad (1)$$

the zonal equation of motion

$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{a \cos \phi \partial \lambda} - v \frac{\partial u}{a \partial \phi} - \omega \frac{\partial u}{\partial p} - \frac{\partial \Phi}{a \cos \phi \partial \lambda} + v \left(f + \frac{u}{a} \tan \phi \right) + F_x \quad (2)$$

and the meridional equation of motion

$$\frac{\partial v}{\partial t} = -u \frac{\partial v}{a \cos \phi \partial \lambda} - v \frac{\partial v}{a \partial \phi} - \omega \frac{\partial v}{\partial p} - \omega \frac{\partial \phi}{\partial \phi} - u \left(f + \frac{u}{a} \tan \phi \right) + F_y \quad (3)$$

The balance equations may be derived as follows:

in the space domain

multiply (1) by $(T/\Theta)^2 c_{\nu} \gamma[\Theta]^{\prime\prime}$ for the balance equation of P_M ;

multiply (1) by $(T/\Theta)^2 c_p \gamma \Theta^*$ for the balance equation of P_E ;

multiply (2) and (3) by [u] and [v], and add for the balance equation of K_M ;

multiply (2) and (3) by u^* and v^* , and add for the balance equation of K_E ;

in the time domain

multiply (1) by $(T/\Theta)^2 c_{\nu} \gamma \overline{\Theta}^{\prime\prime}$ for the balance equation of P_M ;

multiply (1) by $(T/\Theta)^2 c_p \gamma \Theta'$ for the balance equation of P_E ;

multiply (2) and (3) by \overline{u} and \overline{v} , and add for the balance equation of \mathbf{K}_{M} ;

multiply (2) and (3) by u' and v', and add for the balance equation of K_E ;

in the mixed space-time domain

multiply (1) by $(T/\Theta)^2 c_{\nu} \gamma [\overline{\Theta}]^{\prime\prime}$ for the balance equation of \mathcal{P}_{M} ;

multiply (1) by $(T/\Theta)^2 c_p \gamma(\overline{\Theta}^* + \Theta')$ for the balance equation of \mathcal{P}_E ;

multiply (2) and (3) by $[\overline{u}]$ and $[\overline{v}]$, and add for the balance equation of \mathcal{K}_{M} ;

multiply (2) and (3) by $(u'+\overline{u}^*)$ and $(v'+\overline{v}^*)$, and add for the balance equation of \mathcal{H}_E .

Finally, all equations are averaged in time over the period considered and integrated over the total mass of the atmosphere.

A. ENERGY INTEGRALS IN THE SPACE DOMAIN

$$P_{M} = \frac{1}{2}c_{p}\int\overline{\gamma[T]''^{2}}dm$$

$$P_{E} = \frac{1}{2}c_{p}\int\overline{\gamma[T^{*2}]}dm$$

$$K_{M} = \frac{1}{2}\int\overline{([u]^{2}+[v]^{2}})dm$$

$$K_{E} = \frac{1}{2}\int\overline{([u^{*2}]+[v^{*2}]})dm$$

$$G(P_{M}) = \int\overline{\gamma[T]''[Q]''}dm$$

$$G(P_{E}) = \int\overline{\gamma[T^{*}Q^{*}]}dm$$

$$D(K_{M}) = \int\overline{([u][F_{x}]+[v][F_{y}]})dm$$

$$D(K_{E}) = \int\overline{([u^{*}F_{x}^{*}]+[v^{*}F_{y}^{*}]})dm$$

$$P_{M},K_{M}) = -\int\overline{([u]''[\alpha]''}dm = \int f\overline{[u_{g}][v]}dm$$

$$C(P_M, K_M) = -\int [\overline{\omega]''[\alpha]''} dm = \int f[\overline{u_g}][v] dm$$

$$C(P_E, K_E) = -\int \overline{[\omega^*\alpha^*]} dm$$

$$\begin{split} C(K_{E},K_{M}) &= \int [\overline{u^{*}v^{*}}] \cos \phi \, \frac{\eth}{a\eth\phi} \, ([u] \cos^{-1}\phi) dm \\ &+ \int [\overline{u^{*}\omega^{*}}] \, \frac{\eth[u]}{\eth p} \, dm + \int [\overline{v^{*2}}] \, \frac{\eth[v]}{a\eth\phi} \, dm \\ &+ \int [\overline{v^{*}\omega^{*}}] \, \frac{\eth[v]}{\eth p} \, dm - \int [\overline{v}][\overline{u^{*2}}] \, \frac{\tan \phi}{a} \, dm \\ C(P_{M},P_{E}) &= -c_{p} \int \overline{\gamma[v^{*}T^{*}}] \, \frac{\eth[T]}{a\eth\phi} \, dm \\ &- c_{p} \int \overline{\gamma\left(\frac{T}{\Theta}\right)[\omega^{*}T^{*}]''} \, \frac{\eth[\Theta]''}{\eth p} \, dm \end{split}$$

The corrections for the change in time in the balance equations of P_M , P_E , K_M , K_E are, respectively:

$$c_{\rho} \int \overline{\gamma \frac{\partial}{\partial t} \frac{1}{2} [T]''^{2}} dm,$$

$$c_{\rho} \int \overline{\gamma \frac{\partial}{\partial t} \frac{1}{2} [T^{*2}]} dm,$$

$$\int \frac{\overline{\partial}}{\partial t} \frac{1}{2} ([u]^{2} + [v]^{2}) dm,$$

$$\int \frac{\overline{\partial}}{\partial t} \frac{1}{2} ([u^{*2}] + [v^{*2}]) dm.$$

B. ENERGY INTEGRALS IN THE TIME DOMAIN
$$\begin{split} \mathbf{P}_{M} = & \frac{1}{2} \, c_{p} \int \gamma \, [\overline{T}^{\prime\prime}{}^{2}] dm \\ \mathbf{P}_{E} = & \frac{1}{2} \, c_{p} \int \gamma \, [\overline{T}^{\prime\prime}{}^{2}] dm \\ \mathbf{K}_{M} = & \frac{1}{2} \, \int [\overline{u}^{2} + \overline{v}^{2}] dm \\ \mathbf{K}_{E} = & \frac{1}{2} \, \int [\overline{u}^{\prime2} + \overline{v}^{\prime2}] dm \\ G(\mathbf{P}_{M}) = & \int \gamma \, [\overline{T}^{\prime\prime}\overline{Q}^{\prime\prime}] dm \\ G(\mathbf{P}_{E}) = & \int \gamma \, [\overline{T}^{\prime\prime}\overline{Q}^{\prime}] dm \\ D(\mathbf{K}_{M}) = & \int [\overline{u}\overline{F}_{x} + \overline{v}\overline{F}_{y}] dm \\ D(\mathbf{K}_{E}) = & \int [\overline{u}^{\prime\prime}\overline{F}_{x}^{\prime} + \overline{v}^{\prime}\overline{F}_{y}^{\prime}] dm \\ C(\mathbf{P}_{M}, \mathbf{K}_{M}) = & - \int [\overline{\omega}^{\prime\prime}\overline{\alpha}^{\prime\prime}] dm \\ C(\mathbf{P}_{E}, \mathbf{K}_{E}) = & - \int [\overline{\omega}^{\prime\prime}\overline{\alpha}^{\prime\prime}] dm \end{split}$$

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$$C(\mathbf{K}_{E}, \mathbf{K}_{M}) = \int \left[\overline{u'^{2}} \frac{\partial \overline{u}}{a \cos \phi \partial \lambda}\right] dm$$

$$+ \int \left[\overline{u'v'} \cos \phi \frac{\partial}{a \partial \phi} (\overline{u} \cos^{-1} \phi)\right] dm$$

$$+ \int \left[\overline{u'\omega'} \frac{\partial \overline{u}}{\partial p}\right] dm$$

$$+ \int \left[\overline{v'u'} \frac{\partial \overline{v}}{a \cos \phi \partial \lambda}\right] dm$$

$$+ \int \left[\overline{v'^{2}} \frac{\partial \overline{v}}{a \partial \phi}\right] dm + \int \left[\overline{v'\omega'} \frac{\partial \overline{v}}{\partial p}\right] dm$$

$$- \int \left[\overline{u'^{2}} \overline{v} \frac{\tan \phi}{a}\right] dm$$

$$C(\mathbf{P}_{M}, \mathbf{P}_{E}) = -c_{p} \int \gamma \left[\overline{u'T'} \frac{\partial \overline{T}}{a \cos \phi \partial \lambda} \right] dm$$

$$-c_{p} \int \gamma \left[\overline{v'T'} \frac{\partial \overline{T}}{a \partial \phi} \right] dm$$

$$-c_{p} \int \gamma \left[\left(\frac{T}{\Theta} \right) \overline{\omega'T'} \frac{\partial \overline{\Theta'}'}{\partial p} \right] dm$$

The corrections for the change in time in the balance equations of P_M , P_E , K_M , K_E , are, respectively:

$$c_{p} \int \gamma \left[\overline{T}'' \, \overline{\frac{\partial T}{\partial t}}'' \right] dm,$$

$$c_{p} \int \gamma \left[\overline{T}' \, \overline{\frac{\partial T}{\partial t}} \right] dm,$$

$$\int \left[\overline{u} \, \overline{\frac{\partial u}{\partial t}} + \overline{v} \, \overline{\frac{\partial v}{\partial t}} \right] dm,$$

$$\int \left[\overline{u'} \, \overline{\frac{\partial u}{\partial t}} + \overline{v'} \, \overline{\frac{\partial v}{\partial t}} \right] dm.$$

C. ENERGY INTEGRALS IN THE MIXED SPACE-TIME DOMAIN

$$\mathcal{P}_{M} = \frac{1}{2} c_{p} \int \gamma [\overline{T}]^{\prime\prime 2} dm$$

$$\mathcal{P}_{E} = \frac{1}{2} c_{p} \int \gamma [\overline{T^{\prime 2}} + \overline{T}^{*2}] dm$$

$$\mathcal{H}_{M} = \frac{1}{2} \int ([\overline{u}]^{2} + [\overline{v}]^{2}) dm$$

$$\mathcal{H}_{E} = \frac{1}{2} \int [\overline{u^{\prime 2}} + \overline{v^{\prime 2}} + \overline{u}^{*2} + \overline{v}^{*2}] dm$$

$$G(\mathcal{P}_{M}) = \int \gamma [\overline{T}]^{\prime\prime} [\overline{Q}]^{\prime\prime} dm$$

$$G(\mathcal{P}_{E}) = \int \gamma [\overline{T^{\prime}} \overline{Q^{\prime}} + \overline{T}^{*2} \overline{Q}^{*2}] dm$$

$$\begin{split} D(\mathcal{K}_{M}) &= \int ([\overline{u}][\overline{F_{x}}] + [\overline{v}][\overline{F_{y}}]) dm \\ D(\mathcal{K}_{E}) &= \int [\overline{u'F_{x}'} + \overline{v'F_{y}''} + \overline{u}^{*}\overline{F_{x}''} + v^{*}\overline{F_{y}''}] dm \\ C(\mathcal{P}_{M}, \mathcal{K}_{M}) &= -\int [\overline{\omega}]''[\overline{\alpha}]'' dm = \int f[\overline{u_{g}}][\overline{v}] dm \\ C(\mathcal{P}_{E}, \mathcal{K}_{E}) &= -\int [\overline{\omega'\alpha'} + \overline{\omega}^{*}\overline{\alpha}^{*}] dm \\ C(\mathcal{K}_{E}, \mathcal{K}_{M}) &= \int ([\overline{u'v'}] + [\overline{u}^{*}\overline{v}^{*}]) \cos \phi \frac{\partial}{a\partial \phi} ([\overline{u}]\cos^{-1}\phi) dm \\ &+ \int ([\overline{u'\omega'}] + [\overline{u}^{*}\overline{\omega}^{*}]) \frac{\partial [\overline{u}]}{\partial p} dm \\ &+ \int ([\overline{u'v'}] + [\overline{\omega}^{*}\overline{v}^{*}]) \frac{\partial [\overline{v}]}{a\partial \phi} dm \\ &+ \int ([\overline{u'v'}] + [\overline{\omega}^{*}\overline{v}^{*}]) \frac{\partial [\overline{v}]}{\partial p} dm \\ &- \int [\overline{v}]([\overline{u''^{2}}] + [\overline{u}^{*2}]) \frac{\tan \phi}{a} dm \\ C(\mathcal{P}_{M}, \mathcal{P}_{E}) &= -c_{r} \int \gamma ([\overline{v'T'}] + [\overline{v}^{*}\overline{T}^{*}]) \frac{\partial [\overline{T}]}{a\partial \phi} dm \\ &= c_{r} \int \gamma \left(\frac{T}{\Theta}\right) ([\overline{u'T'}] + [\overline{\omega}^{*}\overline{T}^{*}])'' \frac{\partial [\overline{\Theta}]''}{\partial p} dm \end{split}$$

The corrections for the change in time in the balance equations of \mathcal{P}_{M} , \mathcal{P}_{E} , \mathcal{K}_{M} , \mathcal{K}_{E} are, respectively:

$$c_{p} \int \gamma [\overline{T}]^{\prime\prime} \frac{\overline{\partial [T]^{\prime\prime}}}{\partial t} dm,$$

$$c_{p} \int \gamma \left[\overline{T^{\prime}} \frac{\overline{\partial T}}{\partial t} + \overline{T}^{*} \frac{\overline{\partial T^{*}}}{\partial t} \right] dm,$$

$$\int \left([\overline{u}] \frac{\overline{\partial [u]}}{\partial t} + [\overline{v}] \frac{\overline{\partial [v]}}{\partial t} \right) dm,$$

$$\int \left[\overline{u^{\prime}} \frac{\overline{\partial u}}{\partial t} + \overline{v^{\prime}} \frac{\overline{\partial v}}{\partial t} + \overline{u}^{*} \frac{\overline{\partial u^{*}}}{\partial t} + \overline{v}^{*} \frac{\overline{\partial v^{*}}}{\partial t} \right] dm.$$

D. RELATION BETWEEN QUANTITIES IN SPACETDOMAIN (ITALIC CAPITALS) AND QUANTITIES IN MIXED SPACE-TIME DOMAIN (SCRIPT CAPITALS)

$$\begin{split} P_{M} &= \mathcal{P}_{M} + \frac{1}{2}c_{p} \int \gamma \overline{[T]'^{2}} dm \\ P_{E} &= \mathcal{P}_{E} - \frac{1}{2}c_{p} \int \overline{\gamma} \overline{[T]'^{2}} dm \\ K_{M} &= \mathcal{H}_{M} + \frac{1}{2} \int (\overline{[u]'^{2}} + \overline{[v]'^{2}}) dm \end{split}$$

$$K_{E} = \mathcal{H}_{E} - \frac{1}{2} \int (\overline{|u|'^{2}} + \overline{|v|'^{2}}) dm$$

$$G(P_{M}) = G(\mathcal{D}_{M}) + \int \gamma \overline{[T]'[Q]'} dm$$

$$G(P_{E}) = G(\mathcal{D}_{E}) - \int \gamma \overline{[T]'[Q]'} dm$$

$$D(K_{M}) = D(\mathcal{H}_{M}) + \int (\overline{|u|'[F_{Z}]'} + \overline{|v|'[F_{Y}]'}) dm$$

$$D(K_{E}) = D(\mathcal{H}_{E}) - \int (\overline{|u|'[F_{Z}]'} + \overline{|v|'[F_{Y}]'}) dm$$

$$C(P_{M}, K_{M}) = C(\mathcal{D}_{M}, \mathcal{H}_{M}) - \int \overline{|\omega|'[\alpha]'} dm$$

$$C(P_{E}, K_{E}) = C(\mathcal{D}_{E}, \mathcal{H}_{E}) + \int \overline{|\omega|'[\alpha]'} dm$$

$$C(K_{E}, K_{M}) = C(\mathcal{H}_{E}, \mathcal{H}_{M}) + \int \overline{|v^{*}v^{*}|'} \frac{\delta \overline{|v|'}}{a \partial \phi} dm$$

$$+ \int \overline{|u^{*}w^{*}|'} \frac{\delta \overline{|v|'}}{\partial p} dm + \int \overline{|v^{*}v^{*}|'} \frac{\delta \overline{|v|'}}{a \partial \phi} dm$$

$$+ \int \overline{|u|'[v]'} \cos \phi \frac{\delta}{a \partial \phi} (\overline{|u|} \cos^{-1} \phi) dm$$

$$- \int \overline{|u|'[w]'} \frac{\delta \overline{|u|}}{\partial p} dm - \int \overline{|v^{*}v^{*}|'} \frac{\delta \overline{|v|}}{a \partial \phi} dm$$

$$- \int \overline{|u|'[w]'} \frac{\delta \overline{|u|}}{\partial p} dm - \int \overline{|v|'^{2}} \frac{\delta \overline{|v|}}{a \partial \phi} dm$$

$$- \int \overline{|u|'[v]'} \frac{\delta \overline{|v|}}{\partial p} dm + \int \overline{|v|'^{2}} \frac{\delta \overline{|v|}}{a \partial \phi} dm$$

$$- C_{P} \int \gamma \left(\frac{T}{\Theta}\right) \overline{|u^{*}T^{*}|'} \frac{\delta \overline{|O|'}}{a \partial \phi} dm$$

$$+ c_{p} \int \gamma \overline{|v|'[T]'} \frac{\delta \overline{|T|}}{a \partial \phi} dm$$

$$+ c_{p} \int \gamma \overline{|v|'[T]'} \frac{\delta \overline{|T|}}{a \partial \phi} dm$$

$$+ c_{p} \int \gamma \overline{|v|'[T]'} \frac{\delta \overline{|T|}}{a \partial \phi} dm$$

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